

Introducing Guided Mathematical Inquiry in the Classroom: Complexities of Developing Norms of Evidence

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Guided Mathematical Inquiry (GMI) supports the development of deep understandings about mathematics concepts as students learn to address inquiry questions with evidence-based claims. Developing an evidence-based focus has been shown to be problematic with children. This paper presents how a framework for evidence was developed through the expert knowledge of teachers experienced in GMI with components trialled and illustrated using a Year 3 classroom unit on measurement and geometry. This framework can be used to give insight into the complexity of an evidence-focus in mathematics and support further research.

Inquiry-based pedagogies are receiving greater attention in the classroom in Australia (e.g. *reSolve: Mathematics by Inquiry* project - www.resolve.edu.au). What is meant by inquiry is not well-defined in the literature and the term ‘inquiry’ has been used to address a spectrum from Discovery Learning to Theory Improvement (Bereiter & Scardamalia, 1996). To clarify, in the context of this paper, inquiry is considered the addressing of a complex problem in which the means to solving the question is not immediately obvious and in which a single correct solution does not exist. Some inquiry questions used may point towards the mathematical concepts to be addressed, such as “*Can a pyramid have a scalene face?*” or may give no significant insight, such as “*Is Barbie a human?*”. However, in all instances, students are required to explore the question, refine it, obtain mathematical evidence, make a claim and then defend the claim, in a process of argumentation. When addressing such problems, children need to be supported in their learning and thus the term ‘Guided Mathematical Inquiry (GMI)’ is adopted.

In the brief description above of the GMI process, the need for children to work with mathematical evidence to address a question is apparent. However, to engage in inquiry takes a significant pedagogical shift if the adoption of the practice is into a more traditional classroom setting in which the teacher or text has largely been the authority. Learners from classroom settings with these more traditional norms are often accustomed to being told how and when to take a specific approach which will lead to a preferred answer. In such classrooms, reasons and evidence can be overlooked (Driver, Newton, & Osborne, 2000) preventing children from developing behaviours of curiosity, exploration and speaking out (Muller Mirza, Perret-Clermont, Tartas, & Iannaccone, 2009). If this is the case, it can be quite difficult for learners to break these norms and move towards developing their own processes for addressing a mathematical or potentially mathematical question and seeing it through to a reasoned conclusion. As the obtaining, evaluating and presenting of reasoned evidence is at the heart of the inquiry process, one key norm requiring development is that of learners becoming accustomed to working with mathematical evidence.

In the following sections, we address prior research into children’s developing use of evidence in inquiry as well as the notion of classroom norms. We then describe the process by which we developed a framework which illustrates the breadth of evidence usage in GMI. Finally, we refine the framework against a GMI unit to give insight into how an experienced inquiry teacher began to develop evidence usage as a classroom norm.

Literature

Guided Mathematical Inquiry and Evidence

For the purposes of this research, GMI is the addressing of questions that are open-ended and lacking in structure. The latter does not mean that they are poorly worded, but rather that the wording does not clearly point the problem solver to a solution method (Makar, 2012). As the question has neither a single solution process nor a single correct solution, the method of solution and the solution itself need to be justified through the use of evidence. In practice, the evidence put forth may come from the generating of data (surveys, measurements etc), research of existing data, the creation of models, and so forth. However, it is key that the evidence needs to be both appropriate and sufficient to support the claim (Zemal-Saul, McNeill & Hershberger, 2013).

Prior research suggests students experience significant difficulties when working with evidence. For example, when children make an assertion, they tend not to see a need for evidence to support that assertion (Fielding-Wells, 2010; Muller Mirza et al., 2009) often drawing conclusions based on feelings and intuitions rather than evidence (Sampson & Clarke, 2008). Likewise, even when aware of a need for evidence, students may not recognise when they have too little or inaccurate evidence (Zeidler, 1997). Alternatively, students may select evidence according to their ideas and use that evidence to support a pre-determined conclusion rather than drawing the conclusion logically from the evidence (Sampson & Clarke, 2008). Given these difficulties, students require significant support to make a shift to becoming accustomed to the use of evidence when addressing inquiry (Jimenez-Aleixandre & Erduran, 2007).

Sociomathematical Classroom Norms

Classroom norms can be considered the understandings that frame the practices of students and teachers according to what is accepted and valued within a classroom. Such norms are developed and supported by the teacher through ongoing interactions and negotiations within the classroom setting (Goos, 2004). Many norms are learned by children over time, for example, the raising of a hand to ask a question is a broadly encountered school norm. These become norms in that they are habitual to the point of being noticeable when they are not followed. In addressing classroom norms for mathematical inquiry, a distinction can be made between social norms that may exist in the classroom for all discipline areas, and sociomathematical norms which are those specific to the discipline of mathematics (Yackel & Cobb, 1996). A broadly encountered sociomathematical norm is that of valuing the provision of relatively rapid and correct responses, a norm that has been found to be widespread in even early schooling (Franke & Carey, 1996). Once norms such as these are established and reinforced, they do become ingrained and difficult to counter.

The premise behind student-centred pedagogies, such as inquiry, is that students will address more ambiguous questions and devise their own solution methods and means of assessing solutions (Makar, 2012). Thus, a norm of reliance on the teacher to provide this detail is counter to inquiry goals. Sociomathematical norms that are more associated with student-centered practices include the need to support claims, responses and explanations with mathematical reasons and these responses are validated through mathematical argumentation based on evidence (Kazemi & Stipek, 2001).

Existing sociomathematical norms literature addresses norms of classrooms engaged in inquiries involving well-structured, close-ended problems in which a more limited range of methods or approaches are discussed and justified. Questions of the type addressed in this paper are far more complex, requiring students to justify their solution pathway and provide

an argument in which their claim and evidence is opened for evaluation by their peers and teacher. This approach requires that students become practiced in working with evidence, with student decision-making and judgement becoming the norm. There is limited research that addresses how classroom norms are developed that reflect significant shifts towards evidence-based inquiry approaches. Thus, we seek to address the research question: How do experienced inquiry teachers focus students on an evidence-based approach to mathematics?

Development of Evidence Framework

The evidence overview described in this paper came about through two phases of a case study. In the first phase, experienced classroom teachers worked with the researchers to establish an overview of the role of evidence during GMI. In the second phase, this framework was tested against a complete GMI unit as taught by an experienced teacher working with children who were predominantly new to inquiry.

Phase 1

A full-day workshop was held in which seven teachers came together to discuss the classroom norms of GMI. Of these teachers, four had ten or more years of experience teaching with GMI, and three had been teaching using GMI for about one year. The teachers were selected based upon their experience so as to have a mix of teachers who were highly skilled (but may have been teaching GMI for such a period of time that some practices may have become ‘automatic’) and teachers for whom GMI was still new and so could discuss changes to their practice that were fresh in their minds. The teachers were all known to the researchers through their inclusion in ongoing research projects.

The teachers had all been introduced to a Question-Evidence-Conclusion model (Fielding-Wells, 2010) for GMI and were therefore all accustomed to having their students focus on the provision of mathematical evidence as a norm of inquiry. The teachers were asked to consider the ways in which students engaged with mathematical evidence, based across all Inquiries they had undertaken. To do this, they elected to consider each phase of inquiry (Discover, Devise, Develop and Defend: see Allmond, Wells & Makar, 2010), brainstorming all instances of student involvement with evidence during each phase. The researchers led a discussion to question and refine ideas until their nature was clear and categories and subcategories established. As a result, a comprehensive overview of the use of evidence was developed and categorised under the phases of the inquiry: Discover-Envisaging Evidence; Devise-Planning for Evidence; Develop-Generating Evidence; and Defend-Concluding with Evidence.

Phase 2

The first author and two research assistants worked collaboratively to transcribe and code a full GMI unit using the draft framework developed in Phase 1. The unit selected was one taught by one of the experienced project teachers, Mrs Thompson, and was selected because it was the first unit of a school year. The class was comprised of 22 Year 3 students of whom only two were identified as having engaged in some prior Inquiry. The class was unremarkable in that the school is a public school situated in a slightly above average socio-economic area. The class had an approximately even ratio of males to females, several EAL/D students, and a mixed achievement level. The teacher is an experienced primary school teacher with no specific additional training beyond her engagement in a continuing project to develop GMI.

The unit of work selected addressed measurement and geometry by posing the question to the students, “Can you make a one litre container out of paper?”. While the question is

quite structured in terms of directing the students towards some of the mathematics that would be required (capacity), the geometrical concepts required are less obvious – keeping in mind that Year 3 children in Australia have not addressed the relationship between capacity and volume nor the calculation of volume.

In order to trial the draft framework developed, two members of the project team fully coded each section of the unit using substantive codes derived from the framework. At any point when a code did not seem applicable, or could be interpreted in multiple or vague ways, the coding was discussed among the team for clarification, and reworded, removed or separated into multiple codes, to better capture the occurrence under discussion. For example, we became aware when coding that there were instances when students were building 3D models which were not intended to be evidence themselves, but rather to inform their planning by allowing them to visualise the result. Coding these as “*Generating: Organising/Representing Evidence*” was inaccurate as they were more accurately used as prototypes. Thus, a code for “*Planning: Building and/or Trialling a Representation*” was established. The final coding was then reapplied to the entire transcript to ensure that consistency was maintained, amended codes were still applicable to the remainder of the transcript, and that interim changes did not impact previously coded sub-categories. The final (working) framework is provided in Figure 1. While most sub-categories have been allocated to a phase in the inquiry, *Unpacking the Mathematics* was noted to occur across all phases.

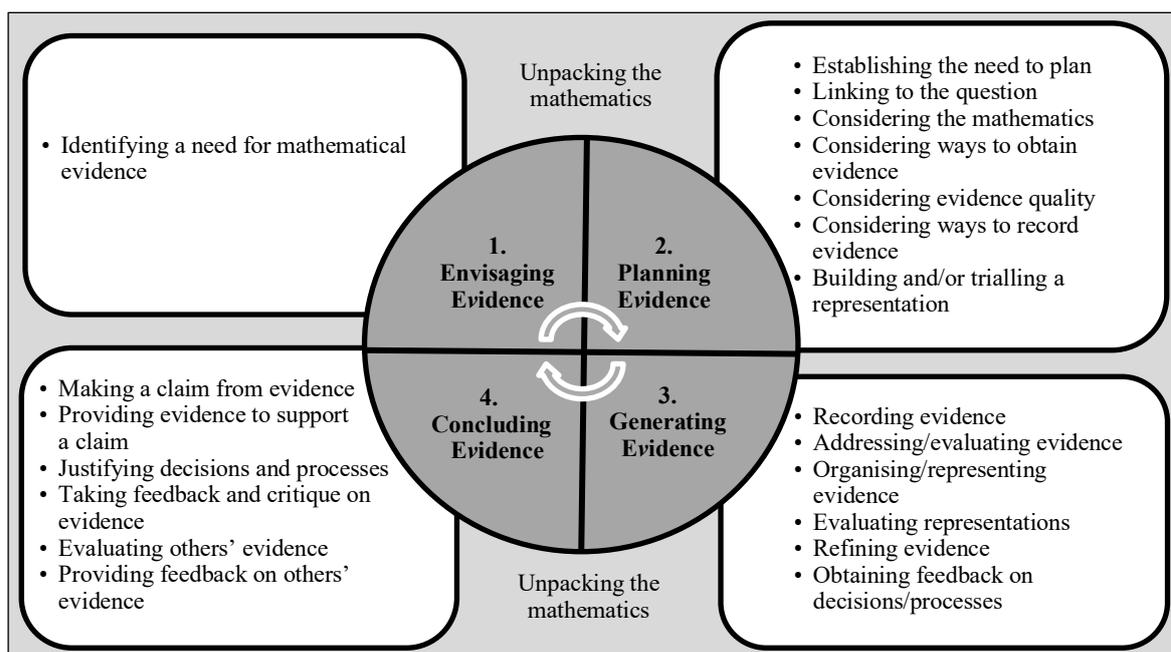


Figure 1. Framework for Evidence focus during Guided Inquiry.

To provide reader background, a summary of the unit follows. While this is not comprehensive, it illustrates the process and provides examples of instances where the sub-category codes (italics) were applied:

Lesson 1 (41 mins) – Envisaging and Planning Evidence: The teacher discussed the nature of inquiry with the students and established that inquiry addresses a question mathematically. She introduced and discussed the question, “Can you make a one litre container out of paper?”, with the students, *unpacking the mathematics* that was relevant by drawing from the children what they thought might be important to know: envisaging a litre, 3D shapes that the container could be made from (e.g. pyramid, rectangular and triangular

prisms, cube), and a discussion of mathematical language (e.g. face, edge, apex, capacity). While not specifically mentioning evidence, the teacher was *identifying a need for mathematical evidence* by recording the mathematical aspects that would be covered.

Towards the end of the session, the teacher asked the students to write down their ideas, in groups, as to how they might make a container, *establishing the need to plan*. The students commenced carrying out their rather ill-formed plans. To do this, students were involved in *building and/or trialling a representation*. For example, one group decided to use a soy milk container they found (one litre based on labelling) and began taping paper around it to get the size they wanted. Groups who finished early began testing their containers in the water trough near the classroom (*building and/or trialling a representation*). The testing was highly inaccurate with water pouring over the edges, miscounts, and use of measuring containers that were thought to hold a litre (e.g. The soy milk container) but which did not.

Lesson 2 (35 mins) – Envisaging and Planning Evidence: The teacher called the students together and introduced the *need for mathematical evidence*: “If you say to me, ‘I just know it’ that’s not good enough..., I need proof and I need evidence.” The students were requested to think about how they might test their evidence more accurately than they have, by *considering ways to obtain evidence* and *evidence quality*. The class discussed possible ways of testing (e.g. pouring sand from a carefully filled one litre container). The children were requested to note their plan for accurate testing in their book before proceeding to the sandpit to test again, further *establishing the need to plan* and *considering evidence quality*. The testing was again highly inaccurate (e.g. containers being squeezed as they were filled, sand pouring over edges). The teacher stopped the class and *considering evidence quality*, raised some of the issues she has seen with measurement before having students identify more. Testing the capacity of the containers continued with increased accuracy and, at student suggestion, multiple remeasures for each container.

Lesson 3 (28 mins) – Planning and Generating Evidence: The teacher revisited the discussion of the previous day *addressing/evaluating evidence* and drew from students their realisation that their construction, measurements and testing had been highly inaccurate. Each container was discussed informally by the group responsible for it, *considering evidence quality* in terms of whether the measuring was accurate, remeasured and so forth. The class discussed ways in which accuracy could be increased. The teacher introduced the need for a conclusion to be drawn and shared how a conclusion draws on the evidence, *making a claim from evidence*. She asked whether students were ready to make a conclusion:

- T: “Question”, “evidence”, can they make a conclusion?
Justine: Not yet [A few other students echo this answer]
T: Not yet. Who agrees, Declan, what do you think? Can they make a conclusion yet?
[Student answers briefly, quietly]. Why?
Declan: Evidence.
T: They don’t have enough ... [Pauses for students to complete the sentence]
Class: Evidence!

Lesson 4 (50 mins) – Planning and Generating Evidence: *Unpacking the mathematics*, the teacher discussed nets with students and demonstrated deconstruction of boxes to show their net. *Obtaining feedback on decisions/processes*, one group shared how they created their container by wrapping paper around a known one litre cube and taping it in place. Discussions as to why this had failed ensued (the wall of the cube was thick and therefore the external dimensions were 10.5cm not 10cm). *Unpacking the mathematics* of nets to construct boxes helped students in *refining evidence* as the group felt a better approach would have been to create a net of a cube of 10cm. The teacher (to prevent all students merely building a 10cm cube) then directed the students to a refined inquiry question: “Can you make a container which holds half a litre? (using a net)”.

Lesson 5 (67 mins) – Planning and Generating Evidence: The students commenced this lesson by planning the container they were going to build. *Establishing the need to plan*, this time the groups were *considering the mathematics* as they drew nets based on the dimensions of geometric shapes in the classroom. One of the six groups (working with a cube) labelled their drawing with measurements (5cm for each side length). *Considering evidence quality*, the students showed greater care with the ruling and measuring sides. As the students work, the teacher discussed with each group what their evidence would be, helping students in *refining evidence* and to *(obtain) feedback on decisions/processes*. After plans were completed, the students moved on to *representing their evidence* by building their designs from their plans.

Lesson 6 (31 mins) – Generating Evidence: The teacher initiated a discussion about the 5cm cube, enabling students to *obtain feedback on decisions/processes*. The group responsible worked out that the cube they had built was not going to hold half a litre. The teacher discussed the relationship between volume and capacity by asking the group who created the 10cm cube (from a litre shape) to share the dimensions of their cube. *Unpacking the mathematics*, she compared the container to a thousands MAB block and drew out the relationship between volume and capacity. The group with the 5cm cube surmised that their cube would hold 125mL and, in checking (*evaluating representations*), found this correct. The class realised the error was in halving all side lengths of the cube rather than just one.

Lesson 7 (32 mins) – Concluding Evidence: The students, in groups, shared their containers with the rest of the class along with the results of their repeated iterations of testing *providing evidence to support a claim*. The majority had made containers which did not hold exactly 500mL but the students were able to clarify the issues and propose a way they would improve their next iteration if they were to continue, *evaluating evidence*.

In this unit of work, the teacher was introducing the children to inquiry for predominantly the first time. She chose to develop familiarity with some key aspects of inquiry: the question-evidence-conclusion link and the need for evidence to be accurate and sufficient. The students did not find this easy, requiring several iterations to develop a need for accuracy and ultimately making the conclusion that they had not accurately made the requisite containers. However, through the inquiry above, the students were able to develop a more robust conceptual understanding of aspects of geometry and measurement: they developed a referent benchmark for a litre; made links between 3D shapes and their nets; and made connections between volume and capacity. These are key conceptual understandings children require to continue to more complex concepts. Procedurally, some students addressed formulaic approaches to calculating volume; however, this was not across the board. The students also demonstrated developing epistemic knowledge – knowledge of acceptable practice across the discipline, through showing their beginning appreciation for evidence-based responses in which evidence is deemed both accurate and sufficient. Multiple iterations would be required for this early conception to develop into a norm.

Discussion

Given the strength of prior research that suggests students have trouble developing an evidentiary focus when addressing inquiry (e.g. Jimenez-Alexandre & Erduran, 2007; Muller Mirza et al., 2009; Sampson & Clarke, 2008), we perceived a need to develop an understanding of how experienced teachers of mathematical inquiry develop focus on evidence as a sociomathematical norm with their students. To do so, we drew on the expertise of those teachers to share their experience in identifying the way in which children were exposed to and accustomed to using evidence during an inquiry, and then used that information to develop a framework which was trialled and refined against an inquiry unit of work. The unit of work selected addressed a class of children novice to inquiry to identify

the introduction of the practices intended to become sociomathematical norms. As this was an introductory unit, there are anticipated differences from subsequent units which should be mentioned as there are implications for the framework development. First, the teacher moved between the planning and generating stages repeatedly to facilitate student guidance. Rather than having students plan the evidence they needed, including how to gather and evaluate their evidence in one move, she shifted through these phases in cycles to provide additional learner support and to reinforce the need for evidence and evidence quality. A second note is that most of the experienced teachers' inquiries result in students formally presenting their findings in oral and/or written format, clearly stating their claim and evidence. Again, the teacher modified practice to remove a more daunting aspect in the early stages. Finally, due to the introductory nature of this unit, not all codes were able to be assigned, however, these were maintained as identified by expert teachers as important.

In line with previous research demonstrating the necessity for student support when learning to take an evidentiary focus (Jimenez-Aleixandre & Erduran, 2007), the students engaged in this inquiry were also supported heavily, and taken through multiple iterations, to develop an appreciation for the need for mathematical evidence when addressing questions. It was however possible to counter the issue of children making assertions without evidence, as previously noted (Muller Mirza et al., 2009). As students engaged in refined creation and testing, they articulated a need for more precise measurement in the creation and testing of their containers. This epistemic knowledge sets the foundation for norm development through subsequent inquiries in which the teacher continues to reinforce and cement this understanding. A key component in the development of this knowledge was the extent to which communication of ideas and language was used in the classroom, with learners being constantly queried whether they had enough, accurate evidence.

Conclusion

The purpose of this aspect of the ongoing research, as reported in this paper, was to address how experienced teachers of GMI, focus students on an evidence-based approach to mathematics. The development of the evidence framework served to identify the breadth of the role of mathematical evidence, which is clearly quite substantial. Not all sub-categories identified are likely to be addressed in every inquiry unit as the teacher selects specific aspects on which to focus. In this instance, we have seen some insight into what aspects of an evidence-based focus an experienced teacher first aims to develop. This focus provides guidance when working with teachers new to inquiry, suggesting this may be introducing students to the broader awareness of a need for evidence in addressing an inquiry question and the need for that evidence to be both sufficient and accurate.

In trialling the framework against this unit, not all subcategories were identified as the students passed through iterative stages of planning and generating evidence rather than focussing on the presentation of a concluding argument. This, along with the application of the framework to only one inquiry unit, is a practical limitation to the research. However, given the input of expert teachers, further changes to the framework are anticipated to be relatively minor. In progressing from here, this framework will be applied to an increased number of GMI units to further refine the framework and to use it to identify and give insight to key evidence components of inquiries, providing further knowledge of the development of evidence norms in the classroom.

Acknowledgements:

This work was supported by funding from the Australian Research Council (DP170101993). The authors wish to gratefully acknowledge the coding contributions of Alinta Brown and Stacey Winch and to thank the teachers engaged in this research.

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